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B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2015
FIRST YEAR
MATHEMATICS (General)
Paper : II

Date : 25/05/2015
Time : 11 am – 2 pm
Full Marks : 75

[Use separate Answer Book for each group]

Group – A

Answer ***any three*** questions from the following : (3 × 5)

1. Find the transformation which transforms the equation $x^2+y^2-2x+14y+20=0$ into $x'^2+y'^2-30=0$. 5
2. If the triangle formed by the straight lines $ax^2+2hxy+by^2=0$ and $lx+my=1$ be right angled, then prove that $(a+b)(al^2+2hlm+bm^2)=0$. 5
3. If the polar of a point with respect to the parabola $y^2 = 4ax$, touches the parabola $x^2 = 4by$, then show that the locus of the point is the hyperbola $xy + 2ab = 0$. 5
4. Show that the straight line $r\cos(\theta-\alpha)=p$ touches the conic $\frac{l}{r}=1+e\cos\theta$, if $(l\cos\alpha - ep)^2+l^2\sin^2\alpha=p^2$. 5
5. Reduce the equation $11x^2-4xy+14y^2-58x-44y+71=0$ to its canonical form and find the nature of the conic represented by it. 4+1

Answer ***any three*** questions from the following : (3 × 5)

6. Prove by vector method $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ in a triangle ABC, where $a=BC$, $b=CA$, $c=AB$. 5
7. (a) Find the projection of the vector $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ along the vector $\vec{b}=4\hat{i}-4\hat{j}+7\hat{k}$. 2

   (b) A particle acted on by constant forces $2\vec{i}+3\vec{j}-\vec{k}$ and $3\vec{i}-\vec{j}+5\vec{k}$ is displaced from the point $A(1,3,2)$ to the point $B(4,5,–1)$, find the work done by the forces. 3
8. Prove that $\left[\vec{\alpha}\times\vec{\beta}\ \ \vec{\beta}\times\vec{\gamma}\ \ \vec{\gamma}\times\vec{\alpha}\right]=\left[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}\right]^2$. 5
9. (a) Find the vector equation of the plane perpendicular to the vector $2\vec{i}+3\vec{j}+6\vec{k}$ and passing through the terminal point of the vector $\vec{i}+5\vec{j}+3\vec{k}$. 4

   (b) If $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-\vec{b}\right|$ and $\left|\vec{a}\right|=3$, $\left|\vec{b}\right|=1$, find $\left|\vec{a}\times\vec{b}\right|$. 1
10. (a) Find the moment of a force $4\vec{i}+2\vec{j}+\vec{k}$ through the point $5\vec{i}+2\vec{j}+4\vec{k}$ about the point $3\vec{i}-\vec{j}+3\vec{k}$. 2

    (b) Find the volume of the tetrahedron $ABCD$ where the position vectors of A,B,C,D are $(-1,1,1),(1,-1,1),(1,1,-1)$ and $(4,1,-3)$ respectively. 3

Group – B

Answer ***any five*** questions from the following : (5 × 5)

11. (a) Let $\{x_n\},\{y_n\},\{z_n\}$ be three sequences of real numbers such that $\lim_{n\to\infty} x_n=\lim_{n\to\infty} z_n=l\ (l\in\mathbb{R})$ and $x_n\le y_n\le z_n\,\forall\, n\in\mathbb{N}$. Show that $\lim_{n\to\infty} y_n=l$. 3

    (b) Give example of two sequences $\{x_n\},\{y_n\}$ of real numbers such that $\{x_n y_n\}$ is convergent but none of $\{x_n\}$ and $\{y_n\}$ is convergent. 2

12. Test the convergency of the following infinite series: 5

$$x^2+\frac{2^2}{3.4}.x^4+\frac{2^2.4^2}{3.4.5.6}.x^6+\frac{2^2.4^2.6^2}{3.4.5.6.7.8}.x^8+\ldots$$

13. (a) State Rolle's theorem for a real valued function. 2

(b) Examine the validity of the hypotheses and the conclusion of Rolle's theorem for the function $f(x)=1-x^{\frac{2}{3}}$ on [–1,1]. 3

14. Expand sin x in Maclaurin's infinite series. 5

15. (a) If $\lim\limits_{x\to 0}\frac{\sin 2x+a\sin x}{x^3}$ be finite, find the value of a and it's limit. 2

(b) Show that $\cos x \sin^3 x$ is maximum at $x=\frac{\pi}{3}$. 3

16. Use Lagrange's method of undetermined multipliers to find the stationary point of $x^2+y^2+z^2$ subject to the condition $x+y+z=6$. 5

17. Find the rectilinear asymptotes of the curve $x^3+x^2y-xy^2-y^3+2xy+2y^2-3x+y=0$. 5

18. Find the envelope of the family of straight lines $\frac{x}{a}+\frac{y}{b}=1$, where a, b are parameters connected by the relation $a^2+b^2=c^2$. 5

Group – C

Answer ***any one*** question from the following : (1 × 10)

19. (a) Find the value of $\lim\limits_{n\to\infty}\left[\frac{n}{n^2}+\frac{n}{n^2+2.1^2}+\frac{n}{n^2+2.2^2}+\ldots+\frac{n}{n^2+2(n-1)^2}\right]$. 3

(b) Evaluate : $\int\frac{dx}{13+4\sin x+3\cos x}$. 3

(c) If $I_n=\int_0^{\frac{\pi}{4}}\tan^n x\,dx$, show that $I_n=\frac{1}{n-1}-I_{n-2}$ where $n\in N,\ n>1$. Hence find the value of $\int_0^{\frac{\pi}{4}}\tan^6 x\,dx.$ 2+2

20. (a) If a function f is integrable in a closed interval [0, a], prove that

$$\int_0^a f(x)\,dx=\int_0^{a/2} f(x)\,dx+\int_0^{a/2} f(a-x)\,dx.$$ 2

(b) Evaluate: $\int_0^{\frac{\pi}{2}}\frac{x\,dx}{(\sin x+\cos x)^2}$. 4

(c) Evaluate the integral by Wallis' method of summation: $\int_a^b\sqrt{x}\,dx.$ 4

Answer ***any one*** question from the following : (1 × 10)

21. (a) Solve $(x+y+1)\,dx=(2x+2y+1)\,dy.$ 4

(b) Find the general and singular solution of $y=px+\sqrt{a^2p^2+b^2}$ (a, b are constants). 4+2

22. (a) Examine whether the differential equation $(\sin y+y\cos x)\,dx+(\sin x+x\cos y)\,dy=0$ is exact. Find the general solution of the equation. 2+3

(b) Solve $\frac{dy}{dx}+2xy=e^{-x^2}$ 4

(c) Form the differential equation from the given relation by eliminating the parameters A, B: $xy=Ae^x+Be^{-x}+x^2$ 1

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